SHOCK-INDUCED SPREADING OF A FILM OF A NON-NEWTONIAN LIQUID

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The system of hydrodynamic equations describing the spreading of a thin film of a non-Newtonian liquid between two solid colliding surfaces is solved. The temperature profiles along the film radius and thickness are found under the assumption of adiabatic compression.

The flow of an incompressible liquid of constant viscosity (a Newtonian liquid) between solid colliding plates was analyzed in [1]; the shock-induced heating of such a liquid was analyzed in [2], where the thermal conductivity, the temperature dependence of the viscosity, and the compressibility of the striker material were taken into account.

The hydrodynamic equations describing the spreading of a thin film of a non-Newtonian liquid were integrated numerically in [3] for certain types of shock. The pressure profile in a non-Newtonian film was found in [4], with inertial forces neglected.

In many practical applications, in particular in analyzing the processes involved in stamping, it is important to know how non-Newtonian materials move and to estimate their heating.

Below we are concerned with the case in which an incompressible, non-Newtonian liquid is subjected to shock; we find the temperature increase, taking into account the temperature dependence of the effective viscosity. We also take into account the resulting deformation in the striker system, which greatly limits the pressure rise in the liquid film.

The behavior of a non-Newtonian liquid is frequently described by means of a power law [5]:

$$\tau = m\dot{\varepsilon}^n = m\left|\frac{\partial u}{\partial z}\right|^n \operatorname{sign}\frac{\partial u}{\partial z}, \tag{1.1}$$

where m and n generally depend on the pressure and temperature, m strongly and n extremely weakly.

The apparent viscosity μ_a for a power law can be expressed in terms of n:

$$\mu_a = m\dot{\varepsilon}^{n-1} = m\left|\frac{\partial u}{\partial z}\right|^{n-1}; \tag{1.2}$$

since n < 1 for pseudoplastic materials, μ_a falls off with increasing rate of shear.

We assume that a thin film of a non-Newtonian liquid of thickness δ_0 fills the gap between a rigid support (anvil) and the striker, with a base radius R and length l. The liquid begins to spread when the freely falling weight M collides with the striker.

The system of hydrodynamic equations for the case $\delta_0/R \ll 1$ is written in terms of cylindrical coordinates (Fig. 1) as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + v\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{1}{\rho}\frac{\partial}{\partial z}\left(m\left|\frac{\partial u}{\partial z}\right|^{n-1}\frac{\partial u}{\partial z}\right); \tag{1.3}$$

$$\frac{\partial p}{\partial z} = 0, \quad \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial v}{\partial z} = 0. \tag{1.4}$$

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The boundary conditions for (1.3) and (1.4) are

$$u(r,\ 0,\ t)=u(r,\ \delta,\ t)=0,\ p(R,\ t)=0,\ v(r,\ 0,\ t)=0,\ v(r,\ \delta,\ t)=w. \tag{1.5}$$

We solve Eqs. (1.3) approximately by the method of moments; we write u as the series

$$u=\left[1-\left|1-\frac{2z}{\delta}\right|^{\frac{n+1}{n}}\right]\cdot[f_0(r,\ \delta)+zf_1(r,\ \delta)+\cdots], \tag{1.6}$$

which satisfies the conditions that the liquid velocity vanishes at the boundaries with the anvil and striker, at $z = 0$ and $z = \delta$, respectively.

Restricting the analysis to the zeroth approximation in (1.6), and integrating continuity equation (1.4) over z, we find

$$v=-\frac{1}{r}\frac{\partial}{\partial r}rf_0\int_0^z\left[1-\left|1-\frac{2z}{\delta}\right|^{\frac{n+1}{n}}\right]dz.$$

For the upper half of the film, $1 \le (2z/\delta) \le 2$, we find, using the boundary condition $v(r,\ \delta,\ t) = w$,

$$w=-\frac{1}{r}\frac{\partial}{\partial r}rf_0\left\{\int_0^{\delta/2}\left[1-\left(1-\frac{2z}{\delta}\right)^{\frac{n+1}{n}}\right]dz+\right.$$

$$\left.+\int_{\delta/2}^{z}\left[1-\left(\frac{2z}{\delta}-1\right)^{\frac{n+1}{n}}\right]dz\right\},$$

and thus ($\eta = 2z/\delta$)

$$u=-\frac{2n+1}{2(n+1)}\frac{wr}{\delta}\,[1-|1-\eta|^{\frac{n+1}{n}}], \tag{1.7}$$

$$v_1=\frac{w}{2(n+1)}\{(2n+1)\eta-n[1-(1-\eta)^{\frac{2n+1}{n}}]\},\ 0\leqslant\eta\leqslant 1, \tag{1.8}$$

$$v_2=\frac{w}{2(n+1)}\{(2n+1)\eta-n[1+(\eta-1)^{\frac{2n+1}{n}}]\},\ 1\leqslant\eta\leqslant 2. \tag{1.9}$$

Here the subscript "1" on v means the lower half of the film, and "2" means the upper half.

If the striker system is quite rigid ($k \to \infty$), we can describe the deceleration of the weight by

$$M\frac{dw}{dt}=\langle p\rangle S,\ \ \langle p\rangle=\frac{2\pi}{S}\int_0^R p(r)\,r\,dr,\ S=\pi R^2,$$

$$\delta=\delta_0+\int_0^t w\,dt,\ w(0)=w_0<0. \tag{1.10}$$

Then Eqs. (1.3) become

$$w\frac{dw}{d\delta}\frac{\partial u}{\partial\delta}+w\frac{\partial u}{\partial\delta}+u\frac{\partial u}{\partial r}+\frac{2}{\delta}v\frac{\partial u}{\partial\eta}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+$$

$$+\frac{1}{\rho}\left(\frac{2}{\delta}\right)^{n+1}\frac{\partial}{\partial\eta}\left(m\left|\frac{\partial u}{\partial\eta}\right|^{n-1}\frac{\partial u}{\partial\eta}\right),\ \ \frac{\partial p}{\partial\eta}=0. \tag{1.11}$$

Substituting (1.7) and (1.8) into (1.11), after it has been multipled by η^N ($N = 0,\ 1,\ 2,\ \ldots$), and integrating it over η from 0 to 1, we find a system of ordinary differential equations which is equivalent, in the limit $N \to \infty$, to (1.11). Evaluating the zeroth moment, corresponding to angular-momentum conservation, we find

$$\frac{\partial p}{\partial r}=-2m\left(\frac{2n+1}{n}\right)^n\frac{r^n|w|^n}{\delta^{2n+1}}+\frac{\rho rw}{2\delta}\frac{dw}{d\delta}-\frac{3(2n+1)}{3n+2}\frac{\rho rw^2}{2\delta^2}. \tag{1.12}$$

We thus find the pressure profile for m = const to be

$$p=-\frac{2m}{n+1}\left(\frac{2n+1}{n}\right)^n\frac{|w|^n}{\delta^{2n+1}}(r^{n+1}-R^{n+1})+\frac{\rho w}{4\delta}\frac{dw}{d\delta}(r^2-R^2)-\frac{3(2n+1)}{3n+2}\frac{\rho w^2}{4\delta^2}(r^2-R^2), \tag{1.13}$$

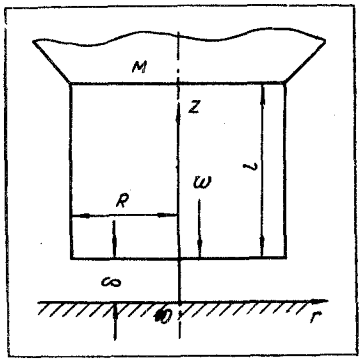

Fig. 1. Physical model and coordinate system.

while the average pressure in the liquid film is

$$\langle p \rangle = \frac{2m}{n+3}\left(\frac{2n+1}{n}\right)^n \frac{|w|^n R^{n+1}}{\delta^{2n+1}} - \frac{\rho w R^2}{8\delta}\frac{dw}{d\delta} + \frac{3}{8}\frac{2n+1}{3n+2}\frac{\rho w^2 R^2}{\delta^2}. \qquad (1.14)$$

If we omit the last two terms from (1.13), corresponding to inertial forces, we find the solution of [4].

In the two limits n = 1 (a Newtonian liquid, $m = \mu$) and n = 0 (an ideally plastic object, $m = \tau_0$), we find from (1.14)

$$\langle p \rangle = -\frac{3\mu w R^2}{2\delta^3} - \frac{\rho w R^2}{8\delta}\frac{dw}{d\delta} + \frac{9}{40}\frac{\rho w^2 R^2}{\delta^2} \quad \text{for } n = 1, \qquad (1.14a)$$

$$\langle p \rangle = \frac{2\tau_0 R}{3\delta} + \frac{3}{16}\frac{\rho w^2 R^2}{\delta^2} - \frac{\rho w R^2}{8\delta}\frac{dw}{d\delta} \quad \text{for } n = 0. \qquad (1.14b)$$

We note that Eq. (1.14a) was derived previously, in [1], while Eq. (1.14b) without the inertial terms is the familiar equation of the theory of plasticity [6] [within a small term Y, which must be introduced because of the change in boundary condition (1.5): p(R, t) = Y].

The average pressure in the liquid film, $\langle p \rangle(\delta)$, cannot be found for the general case, so we turn to two particular flow regimes — the inertial regime, for which the Reynolds number is $Re = \rho|w|\delta/2\mu_a > 1$, and the viscous regime, for which Re < 1. The first regime is usually achieved in the initial stages of high-velocity collisions, while the second is achieved in the final stages, when the liquid is thinner and the striker has been decelerated.

In the case Re ≫ 1 we find from (1.10) and (1.14)

$$M\frac{dw}{dt} = S\left(-\frac{\rho w R^2}{8\delta}\frac{dw}{d\delta} + \frac{3}{8}\frac{2n+1}{3n+2}\frac{\rho w^2 R^2}{\delta^2}\right).$$

Solving this equation, we find

$$w = w_0\left(\frac{1+\lambda_0}{1+\lambda}\right)^{\frac{3(2n+1)}{3n+2}}, \quad \lambda = \frac{\rho R^2 S}{8M\delta}, \quad \lambda_0 = \frac{\rho R^2 S}{8M\delta_0},$$

$$\langle p \rangle = \frac{9}{40}\frac{\rho w^2 R^2}{\delta^2}\frac{3(3n+2) - \lambda(n-1)}{3(3n+2)(1+\lambda)}. \qquad (1.15)$$

In the case Re ≪ 1 we find from (1.10) and (1.14)

$$M\frac{dw}{dt} = -\frac{2mS}{n+3}\left(\frac{2n+1}{n}\right)^n \frac{R^{n+1}|w|^n}{\delta^{2n+1}},$$

from which we find

$$w = w_0\left[1 - \frac{1}{\psi}(\xi - 1)\right]^{\frac{1}{2-n}}, \quad \xi = \left(\frac{\delta_0}{\delta}\right)^{2n},$$

$$\psi = \frac{n+3}{2-n}\left(\frac{n}{2n+1}\right)^n \frac{Mn\delta_0^{2n}|w_0|^{2-n}}{\pi m R^{n+3}}, \qquad (1.16)$$

$$\langle p \rangle = \frac{2m}{n+3}\left(\frac{2n+1}{n}\right)^n \frac{R^{n+1}|w|^n}{\delta_0^{2n+1}}\xi^{\frac{2n+1}{2n}}.$$

If the deformation of the striker is taken into account, the weight decelerates according to a different law. If the film is initially thick, the striker compresses the liquid with essentially no resistance, but as the film becomes thinner the resistance to the striker motion increases rapidly, $\sim\delta^2$ according to (1.15) or $\sim\delta^{2n+1}$ according to (1.16). Accordingly, beginning at some time t_1, the energy of the weight is expended not only on compressing the liquid but also on deforming the striker itself, which is abruptly decelerated and acquires energy which results in the subsequent recoil of the weight. These events prevent the pressure increase in the liquid from becoming too large, as follows formally from (1.15) or (1.16), and they impose a restriction on the duration of the collision itself. The time $t = t_1$ is determined from the condition that the rigidities of the striker system and the liquid film be equal [2]:

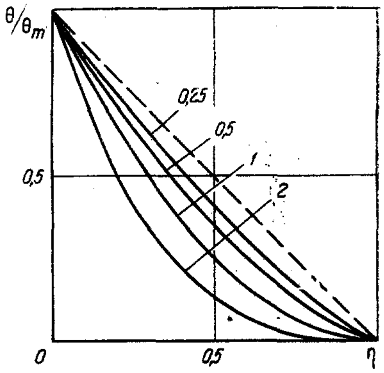

Fig. 2. Temperature profile over the thickness of the liquid film.

$$S\frac{d\langle p\rangle}{d\delta}=-k,\quad 1/k=\sum_i l_i/E_iS_i, \tag{1.17}$$

where E_i is the Young's modulus of element i of the striker system.

To find the position of the characteristic boundary of the initial stage of the collision (in which the striker undergoes essentially no deformation) we consider the two flow regimes in turn.

First, for Re > 1, we find from (1.15) and (1.17)

$$\frac{3\rho R^2w^2S}{40\delta^3(3n+2)^2(1+\lambda)^2}\{\lambda(9n+4)-2(3n+2)]\cdot$$
$$\cdot[3(3n+2)-\lambda(n-1)]-\lambda(n-1)(3n+2)(1+\lambda)\}=-k. \tag{1.18}$$

For small values of λ and λ_0 this equation can be simplified, and an equation can be found for the film thickness:

$$\delta_1\approx\sqrt[3]{\frac{9\pi\rho R^4w_0^2}{20k}}, \tag{1.19}$$

independent of n and δ_0. The independence is attributable to the sharp increase in pressure and the appearance of large gradients in the liquid flow velocity only for sufficiently thin films.

From (1.16)and (1.17) we can determine the characteristic film thickness for the case Re < 1:

$$\alpha=\xi^{\frac{n+1}{n}}\left[(2n+1)(1+\psi-\xi)^{\frac{n}{2-n}}-\frac{2n^2}{2-n}\xi(1+\psi-\xi)^{\frac{2(n-1)}{2-n}}\right],$$
$$\alpha=\frac{n+3}{2\pi}\left[\frac{n(n+3)}{\pi(2-n)}\left(\frac{n}{2n+1}\right)^2\right]^{\frac{n}{2-n}}\left[\frac{M^nk^{2-n}\delta_0^{2(n+2)}}{m^2R^{2(n+3)}}\right]^{\frac{1}{2-n}}. \tag{1.20}$$

If we assume that, after time t_1, the compression of the film occurs with some constant force p_xS, which acts over a time t_x, then we can easily find the deceleration law of the "compressible" striker and determine the limiting thickness δ_k. The quantities p_x and t_x must depend on the energy of the weight, the parameters of the striker system, and the properties of the liquid. In the case of a collision with the anvil, without the liquid, these quantities are determined from [7]

$$p_x=-w_0\sqrt{\frac{kM}{S^2}},\quad t_x=\pi\sqrt{\frac{M}{k}}. \tag{1.21}$$

Turning now to the case Re < 1, and using (1.14), we find

$$\frac{2m}{n+3}\left(\frac{2n+1}{n}\right)^n\frac{R^{n+1}|w|^n}{\delta^{2n+1}}=p_x,$$

whose solution yields

$$w=\frac{d\delta}{dt}=-\left(\frac{n+3}{2m}\right)^{\frac{1}{n}}\frac{np_x^{1/n}}{2n+1}\frac{\delta^{(2n+1)/n}}{R^{(n+1)/n}}, \tag{1.22}$$

$$\beta=\frac{\delta_1}{\delta_k}=\left[1+\frac{n+1}{2n+1}\left(\frac{n+3}{2m}p_x\frac{\delta_1^{n+1}}{R^{n+1}}t_x^n\right)^{1/n}\right]^{\frac{n}{n+1}}. \tag{1.23}$$

The quantity δ_1 is determined from (1.18)-(1.20) for various values of Re. We note that if condition (1.17) holds at the very beginning of the collision, then we have $t_1 = 0$ and $\delta_1 = \delta_0$.

In the case of adiabatic heating of the liquid (with a thermal diffusivity $\varkappa_0 = 0$) the energy dissipated in a Lagrangian particle at coordinates r_0 and z_0 is conserved, so that

$$\frac{DT}{Dt}=\frac{m}{\rho c_v}\left|\frac{\partial u}{\partial z}\right|^{n+1},\quad T(r_0,z_0,0)=0,$$
$$\left(\frac{\partial r}{\partial t}\right)_{r_0,z_0}=u,\quad\left(\frac{\partial z}{\partial t}\right)_{r_0,z_0}=v, \tag{2.1}$$
$$r(r_0,z_0,0)=r_0,\quad z(r_0,z_0,0)=z_0.$$

If the temperature dependence of m is specified over some T range by

$$m = m_0/(1+\gamma T)^\nu, \quad \gamma > 0, \quad \nu > 0, \tag{2.2}$$

then, multiplying (2.1) by $(1+\gamma T)^\nu$ and introducing the temperature function

$$\Theta = [(1+\gamma T)^{\nu+1} - 1]/\gamma(\nu+1), \tag{2.3}$$

we can convert this equation to

$$\frac{D\Theta}{Dt} = \frac{m_0}{\rho c_p}\left|\frac{\partial u}{\partial z}\right|^{n+1}. \tag{2.4}$$

Equation (2.4) differs from (2.1) in that T is replaced by Θ (in the case $\nu = 0$ we have $T = \Theta$).

To establish the relation between the running coordinates and the Lagrangian coordinates, we write the system of differential equations of the vector lines as

$$\frac{dr}{u} = \frac{dz}{v} = \frac{d\delta}{w}. \tag{2.5}$$

Using (1.7)-(1.9), we find from (2.5)

$$z = \frac{\delta}{2}\left[1 \mp \left(\frac{\delta}{\delta+c}\right)^{\frac{n}{n+1}}\right] \quad \begin{pmatrix} - \text{ for } & 0 \leqslant z \leqslant \delta/2 \\ + \text{ for } & \delta/2 \leqslant z \leqslant \delta \end{pmatrix},$$

$$r = r_0\left[\frac{\delta_0(\delta+c)}{\delta(\delta_0+c)}\right]^{(2n+1)/2(n+1)}, \tag{2.6}$$

$$c = \delta_0\left[1 - \left(1 - \frac{2z_0}{\delta_0}\right)^{\frac{n+1}{n}}\right] \Big/ \left(1 - \frac{2z_0}{\delta_0}\right)^{\frac{n+1}{n}}.$$

We see without difficulty that most of the heating of the liquid must occur during the second stage of the collision, for which we can use the approximation that the compression is achieved by a constant force $p_x S$. Transforming from t to δ in (2.4), and using (1.22) and (1.7), we find the liquid temperature for $0 \leq z \leq \delta/2$ to be

$$\Theta = -\frac{(2n+1)(n+3)}{2n}\frac{p_x}{\rho c_p R^{n+1}}\int_{\delta_1}^{\delta_k}\frac{r^{n+1}}{\delta}\left(1 - \frac{2z}{\delta}\right)^{n+1} d\delta. \tag{2.7}$$

It follows from (2.7) that the maximum temperature Θ_m is reached at point $r = R$ at $z = 0$:

$$\Theta_m = \frac{(2n+1)(n+3)}{2n} T_x \ln\beta, \quad T_x = \frac{p_x}{\rho c_p}. \tag{2.8}$$

Denoting the Lagrange coordinates (2.6) at time t_1 by a subscript "1," we can evaluate the integral in (2.7) and determine the temperature fields:

$$\Theta = \frac{(2n+1)(n+3)}{2n} T_x \left(\frac{r}{R}\right)^{n+1} q^{-(2n+1)}\left[2(q-q_1) + \ln\frac{q-1}{q+1}\frac{q_1+1}{q_1-1}\right],$$

$$q = \left(1 - \frac{2z}{\delta}\right)^{-\frac{n+1}{2n}}, \quad q_1 = \left(1 - \frac{2z}{\delta_1}\right)^{-\frac{n+1}{2n}}. \tag{2.9}$$

In the case n = 1 we find from (2.9) the following equations for the temperature field and the maximum liquid temperature:

$$\Theta = 6T_x\left(\frac{r}{R}\right)^2(1-\eta)^3\left[\frac{2\eta(\beta-1)}{(1-\eta)(\beta-\eta)} + \ln\frac{2\beta-\eta}{2-\eta}\right].$$

$$\Theta_m = 6T_x \ln\beta, \tag{2.9a}$$

These equations are the same as those found previously in [2].

In the other limit, n = 0, we find

$$\Theta_m = \frac{3}{2}T_x \ln 2 \quad \text{for} \quad p_x = \frac{2\tau_0 R}{3\delta_1}, \quad \Theta_m = 0 \quad \text{for} \quad p_x < \frac{2\tau_0 R}{3\delta_1}. \tag{2.9b}$$

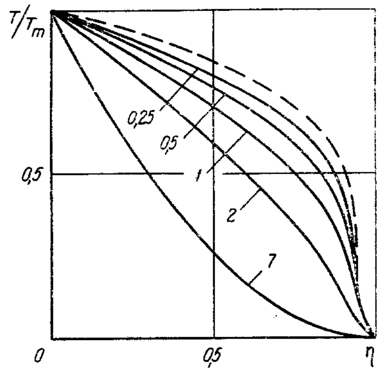

Fig. 3

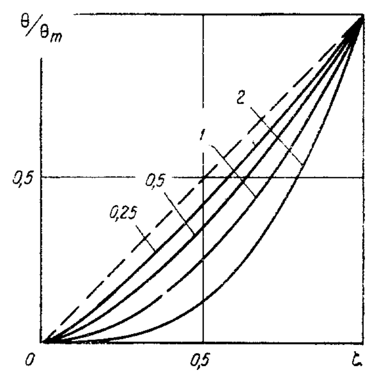

Fig. 4.

Fig. 3. Temperature profile over the thickness of the film; the temperature dependence of the viscosity is taken into account.

Fig. 4. Temperature profiles along the striker radius.

The liquid temperature T is determined from (2.9) simply on the basis of Eq. (2.3).

Figure 2 shows the temperature Θ/Θ_m for the case $r = R$ as a function of the parameter $\eta = 2z/\delta_k$, for $\ln\beta = 2$ and for various values of n: 0.25, 0.5, 1, and 2 (the dashed line is the asymptotic limit for $n \to 0$).

These curves can be described well by a power function $(1-\eta)^{n+1}$, which is convenient for estimating the liquid temperature during the collision. From (2.3) we find

$$T = \frac{1}{\gamma}\{[1 + \gamma(\nu + 1)\Theta]^{\frac{1}{\nu+1}} - 1\}.$$

Accordingly, for sufficiently large Θ we can use the approximation

$$\frac{T}{T_m} \approx \left(\frac{\Theta}{\Theta_m}\right)^{\frac{1}{\nu+1}} = (1-\eta)^{\frac{n+1}{\nu+1}}. \tag{2.10}$$

Figure 3 shows curves of the temperature T/T_m for the cases $n = 0.25$, 0.5, 1, 2, and 7 with $\nu = 3$, constructed from (2.10); in plotting these curves we used $\partial u/\partial z = 0$ for $\eta = 1$ Eq. (1.7) so that there is no heating at the center of the film. We see from this figure that with $n < \nu$ the temperature dependence of the viscosity, (2.2), leads to an averaging of the heating over the thickness of the film. This result is not a contradiction of the physics of the phenomenon: since there is a sharp temperature gradient in the central part of the film in the case $\varkappa_0 \neq 0$, a heat flux directed toward the center can arise. This heat flux also facilitates an equalization of temperatures. In the case $n > \nu$ the nature of the temperature profiles does not change.

Figure 4 shows the temperature profile along the striker radius, $\xi = r/R$, for $\eta = 0$ and for various values of n.

NOTATION

r, z	are the axes of the cylindrical coordinate system;
u, v	are the radial and axial velocity components of the liquid;
p	is the pressure;
$\langle p \rangle$	is the average pressure;
$\rho, c_p, \varkappa_0$	are the density, specific heat, and thermal diffusivity of the liquid;
l, R, S	are the length, radius, and base area of the striker;
E	is the Young's modulus of the striker material;
k	is the rigidity of the striker system;
i	is the number of elements in the striker system;
M	is the mass of the falling weight;
δ_0	is the initial thickness of the liquid film;
δ	is the thickness at time t;
δ_k	is the limiting film thickness;

w_0	is the initial striker velocity;
w	is the striker velocity at time t;
t_1	is the beginning of the second stage of the collision;
t_x	is the duration of this stage;
p_x	is the pressure during the collision;
δ_1	is the film thickness at time t_1;
η	is the dimensionless film thickness;
ζ	is the dimensionless radius;
$\dot{\varepsilon} = \partial u/\partial z$	is the rate of shear;
τ	is the tangential stress;
τ_0	is the yield point for pure shear;
Y	is the yield point for uniaxial stress;
m, n, γ, ν	are the rheological constants of the liquid;
m_0	is the value of m at normal temperature;
μ	is the dynamic viscosity of liquid;
μ_a	is the effective viscosity of liquid;
T, T_x, and Θ	are the liquid temperature;
T_m and Θ_m	are the maximum temperature;
ψ, ξ, β, q, and c	are the dimensionless quantities;
Re	is the Reynolds number.

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